Influence of Uncertainty and Numerical Errors in the Context of Broadband MIMO Systems

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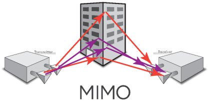
Introduction: MIMO Systems

MIMO:	**M**ultiple **I**nput **M**ultiple **O**utput

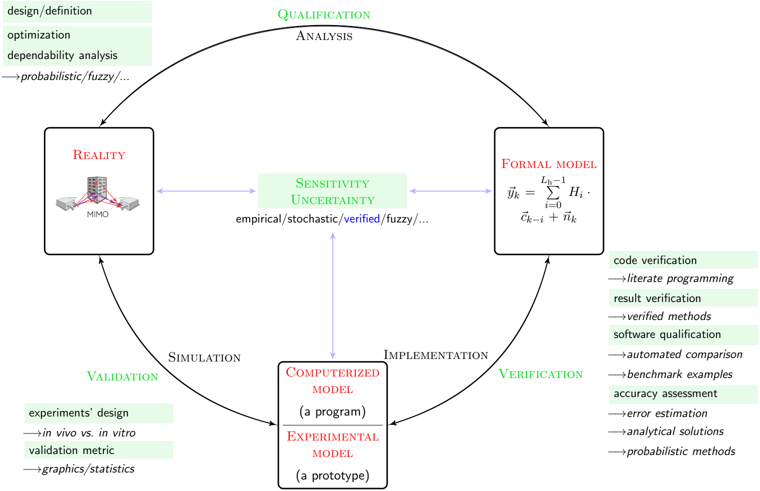


Increase the channel capacity without increasing the channel bandwidth or the transmit power

Method:	Multiple data streams are transmitted on the same frequency band and at the same time
Separation:	Spatial, for example, multiple antennas at the transmitter and receiver side at different locations
Channel capacity:	The information theoretic limit on the bit rate (BER)
BER:	The number of bits per second that can be transmitted through a physical channel error free

All stages might be affected by uncertainty and numerical errors!

Modeling and Simulation Cycle



High requirements on safety and reliability → Need for V&V

Traditional V&V: A Short Overview

Beginnings: In the area of computational fluid dynamics

Standards: No true standard, only *guidelines*
Software V&V: IEEE 1012

Approaches: Formal methods for mission-critical tools
Syntactic methods otherwise

Necessary:

- Flexible and interoperable data types and libraries
- Adaptive self-learning algorithms
- Dealing with uncertainty or bad/missing data
- Uncertainty visualization
- Distributed and parallel computing, crowd sourcing
- User support...

Our goal: Apply V&V based on result verification to MIMO!

Verification Degree

From the lowest to the highest verification degree:

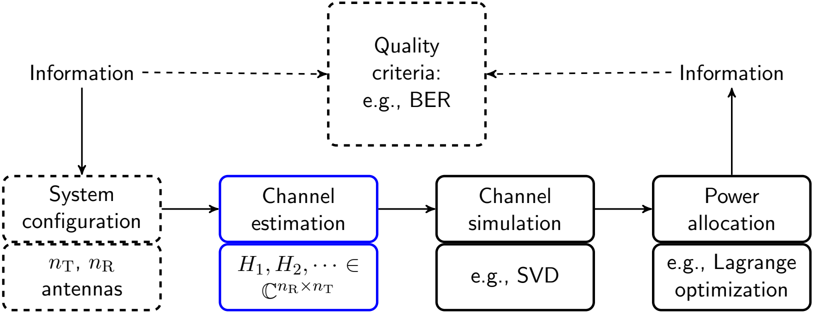
- C4 Standard floating (fixed) point arithmetic
- C3 IEEE 754 arithmetic, traditional sensitivity (e.g. Monte-Carlo)
- C2 Subsystems verified
- C1 The whole process verified (IEEE 754/P1788)

Additionally alongside the degree:

- $+$ code verification
- $-$ no uncertainty/sensitivity analysis
- $Q_{\text{id}}q$ a quality indicator, $q \in [0, 1]$, id$\in \{\underline{n}ominal, \underline{u}ncertain\}$

Questionnaires developed to establish the V&V degree

Subprocesses for a (MIMO) Digital Channel



System: Frequency selective (n_T transmitting, n_R receiving antennas)

Model: $\vec{y}_k = \sum_{i=0}^{L_h-1} H_i \cdot \vec{c}_{k-i} + \vec{n}_k,\ \vec{y}_k, \vec{n}_k \in \mathbb{C}^{n_R},\ \vec{c}_{k-i} \in \mathbb{C}^{n_T},\ H_i \in \mathbb{C}^{n_R \times n_T}$

where: $k \in \mathbb{Z}$ time; L_h the number of channel taps; $\vec{y}_k$ received data; $\vec{c}_{k-i}$ transmitted signal; $\vec{n}_k$ receiver noise (variance σ); H_i channel matrix

Channel Estimation: Questionnaire

Goal: Identify the values of the channel matrices

Four areas:

- Description of input data
- Description of models
- Description of algorithms
- Description of output data

Practice mode: Implementation on an Application Specific Integrated Circuit (ASIC) or a Digital Signal Processor (DSP)

$\rightarrow$ Use fractional/fixed point number format to store data and carry out computations

Simulation mode: Testing of the developed algorithms

$\rightarrow$ PC simulation is sufficient

Former verification degree: C4- (practice), C3 (simulation)

Description of models (1)

Important for: Establishing the verification degree

Goal: Describe the interconnection between the received and transmitted signals

Formulas: $\vec{r}_\nu = (S_1 \; S_2 \; \dots \; S_{n_\mathrm{T}}) \cdot \begin{pmatrix} \vec{h}_{\nu 1} \\ \vdots \\ \vec{h}_{\nu n_\mathrm{T}} \end{pmatrix} + \vec{n}_\nu$ or $\vec{r}_\nu = S\vec{h}_\nu + \vec{n}_\nu$

where

- S pilot sequence vectors
- $\vec{r}_\nu$ the received signal for each output
- $\vec{h}_{\nu\mu} \in \mathbb{C}^{L_\mathrm{h}}$ channel taps between the MIMO input μ and the output ν
- $L_\mathrm{h} \approx$ the number of matrices H_i to be estimated

Practicality condition: $L_\mathrm{s} - L_\mathrm{h} + 1 \geq n_\mathrm{T} L_\mathrm{h}$

Minimum pilot sequence length: $L_\mathrm{s} = n_\mathrm{T} L_\mathrm{h} + L_\mathrm{h} - 1$

Description of models (2)

Unknowns: $$H_i = \begin{pmatrix} \mathbf{h}_{11}[i] & \cdots & \mathbf{h}_{1\mathrm{n_T}}[i] \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{\mathrm{n_R}1}[i] & \cdots & \mathbf{h}_{\mathrm{n_R n_T}}[i] \end{pmatrix}$$

Solution by the least squares minimization: $\vec{h}_\nu = (S^{\mathsf{H}}S)^{-1}S^{\mathsf{H}}(\vec{r}_\nu + \vec{n}_\nu)$

Numerically more feasible: $S^{\mathsf{H}}(R+N) = S^{\mathsf{H}}SH$

S with optimality: $\vec{h}_\nu = \frac{1}{(L_\mathrm{s}-L_\mathrm{h}+1)U_\mathrm{s}^2} \cdot S^{\mathsf{H}}\left(\vec{r}_\nu + \vec{n}_\nu\right)$

Advantageous under noise: Choose pilot sequences with higher lengths

But: Higher pilot lengths reduce the capacity for transmitting useful info

Parameters:

- $n_\mathrm{T} > 1$, $n_\mathrm{R} > 1$ (typically 2,3,4).
- L_s depends on the application (e.g., 5, 33, 321)
- S analytic to fulfill $S^{\mathsf{H}}S = \alpha I$ or random

Description of Input Data

Important for: Establishing tolerance of measurements

Input quantities:

- n_{T}, n_{R}, L_{h} (integer numbers)
- pilot sequences S from a look-up table
- the received data R from an analog to digital converter

Source: Oscilloscope or computer simulation

Description: Hierarchical Data Format (HDF5), ASCII data file

Pre-selection: none

Accuracy:

- Exact for the pilot sequences
- Discretized by the ADC, disturbed by noise for received data ($\rightsquigarrow$ intervals possible)

Representation: Fractional/fixed point; double IEEE 754 type

Description of Algorithms

Important for: Establishing the verification degree

Type: Numeric for linear systems; stochastic Monte Carlo simulation for noise (conventional)

$\rightarrow$ Result verification possible

Parallelization: Parallelisable (e.g., computation of channel coefficients for different MIMO outputs)

Architecture: ASIC or DSP (practice mode); a standard PC (simulation mode)

Operations: (Matrix) addition, multiplication + transpose, complex conjugation (arbitrary pilot sequence)

Sub-algorithms: Least squares solution for overdetermined linear systems (arbitrary pilot sequence)

Sensitivity: Numerics

UML description: Omitted

Description of Output Data

Important for: Validation

Depends on: The kind of arithmetic

Stored: Estimated coefficients of H_i

Data type: IEEE double format for the simulation mode, fixed point format for the practice mode (unitless)

Accuracy: $\frac{1}{n_\mathrm{T}\, n_\mathrm{R}\, L_\mathrm{h}} \cdot \mathrm{E}\left\{ \sum_{\nu=1}^{n_\mathrm{R}} \left(\widehat{\mathbf{h}}_\nu - \vec{h}_\nu \right)^\mathrm{H} \cdot \left(\widehat{\mathbf{h}}_\nu - \vec{h}_\nu \right) \right\}$ or

$\mathrm{MSE}_\mathrm{LS} = \frac{\sigma^2}{P_\mathrm{s}} \cdot \frac{n_\mathrm{T}}{L_\mathrm{s} - (L_\mathrm{h} - 1)}$

High accuracy: The higher the length L_s

Failures: Robust aside from the usual numerics failures

Information exchange: Text file with decimal numbers (15 digits, simulation); memory's fixed point format (practice); no plots

Further use: For channel simulation

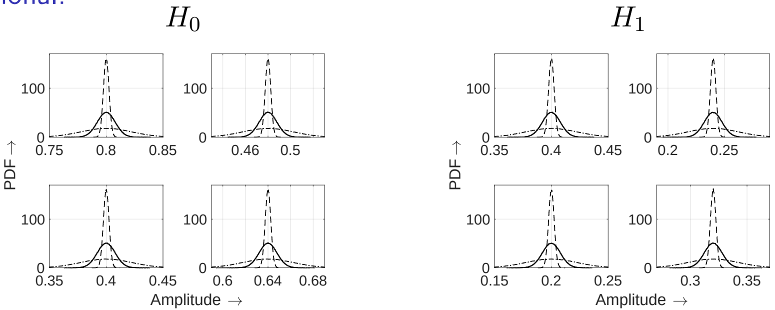
Toy Example: Traditional Channel Estimation

System: Two tap frequency selective (2×2) MIMO channel

$\rightarrow$ $L_\mathrm{h} = 2$, $n_\mathrm{R} = n_\mathrm{T} = 2$, three L_s, SNR=30 dB

For validation: Known $H_0 = \begin{pmatrix} 0.8 & 0.48 \\ 0.4 & 0.64 \end{pmatrix}$, $H_1 = \begin{pmatrix} 0.4 & 0.24 \\ 0.2 & 0.32 \end{pmatrix}$

Traditional:



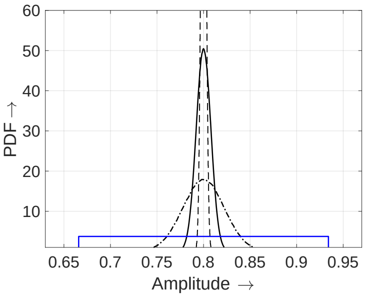
$\rightarrow$ dash-dotted line: $L_\mathrm{s} = 5$, solid line: $L_\mathrm{s} = 33$, dashed line: $L_\mathrm{s} = 321$

Toy Example: Verified Channel Estimation

System: Two tap frequency selective (2×2) MIMO channel

For validation: Known $H_0 = \begin{pmatrix} 0.8 & 0.48 \\ 0.4 & 0.64 \end{pmatrix}$, $H_1 = \begin{pmatrix} 0.4 & 0.24 \\ 0.2 & 0.32 \end{pmatrix}$

Verified: Noise taken into account as an interval of $\pm 3\sigma$ (C-XSC Toolbox)



$H_0(1,1)$, blue line for intervals

Verification degree: C2-

Important: Since there is no classical noise, L_s does not play a role!

V&V for Channel Simulation

Models: For $L_\mathrm{h} = 1$ (frequency flat MIMO link)

Goal: Obtain a number of independent, weighted SISO links

A possibility: Singular value decomposition (SVD), $H = U \cdot \Sigma \cdot V^\mathsf{H}$

$$\vec{u} := U^\mathsf{H} \vec{z} = U^\mathsf{H} \underbrace{\left(U \Sigma V^\mathsf{H}\right)}_{H} V \vec{c} + \underbrace{U^\mathsf{H} \vec{n}}_{:=\vec{w}} = \Sigma \vec{c} + \vec{w}$$

Ideally: Independent, non-interfering layers u_i having unequal weights $\sqrt{\xi_i}$, $u_i = \sqrt{\xi_i} c_i + w_i$

Problems:

- The layers are actually NOT non-interfering
- H (and therefore $\sqrt{\xi_i}$) are uncertain

Verification C3-: Difficult to improve (e.g., U, V meaningless with intervals, principal component analysis?)

Possibilities: Algorithms based on CORDIC or geometric mean decomposition

Verified Power Allocation

Goal: Assign power to layers in such a way as to minimize BER

More information: *Solving the Power Allocation Problem...* in Int. J. Reliability and Safety, 12(1/2), 2018

Approach: Analytical-verified (Lagrange multipliers + X-CSC)

$$J = \frac{2}{\sum_{l=1}^{L} \log_2 M_l} \sum_{l=1}^{L} \left(1 - \frac{1}{\sqrt{M_l}}\right) \cdot \mathrm{erfc}\left(\frac{1}{2\sigma}\sqrt{\frac{3p_l \cdot \xi_l \cdot P_s}{L(M_l - 1)}}\right) + \lambda \left(\sum_{l=1}^{L} p_l - L\right) \to \min$$

where p_l the power allocation parameters; M_l the number of symbols for the layer l; λ the Lagrange multiplier; P_{s} overall transmit power

Overall verification degree: C2

Result: A verified upper bound on the lower bound of the BER

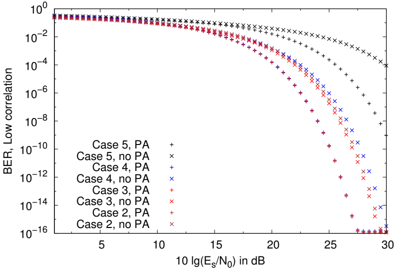
Still needed: A verified upper bound on the BER itself (take the interference between the SISO layers of a MIMO link into account)

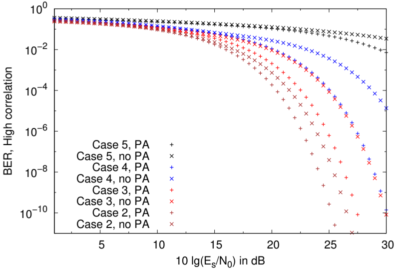
Example

System: Frequency flat with $n_\mathrm{R} = n_\mathrm{T} = 4$, $L = 2, 3, 4$

Low correlation: $\sqrt{\xi_1} \approx 1.751$, $\sqrt{\xi_2} \approx 0.871$, $\sqrt{\xi_3} \approx 0.436$, $\sqrt{\xi_4} \approx 0.214$

High correlation: $\sqrt{\xi_1} \approx 1.903$, $\sqrt{\xi_2} \approx 0.624$, $\sqrt{\xi_3} \approx 0.212$, $\sqrt{\xi_4} \approx 0.0692$





BER is improved in a verified way!

Conclusions

The task of digital MIMO communications analyzed using V&V techniques

Verification degree of two process stages improved, third analyzed

→ Frequency flat and selective MIMO systems considered

Verified techniques could help to

- reduce the pilot sequences' lengths
- take care of numerical errors
- propagate uncertainty

Future work

Test verified channel estimation for real life systems with unknown channels in the simulation mode

Implement interval procedures for the practice mode (⇝ ASIC)

Take into account uncertainty at the channel simulation stage

Characterize and take into account interference between the layers