Worst-case Solution Spaces for Systems Design under Uncertainties

REC Conference 2018

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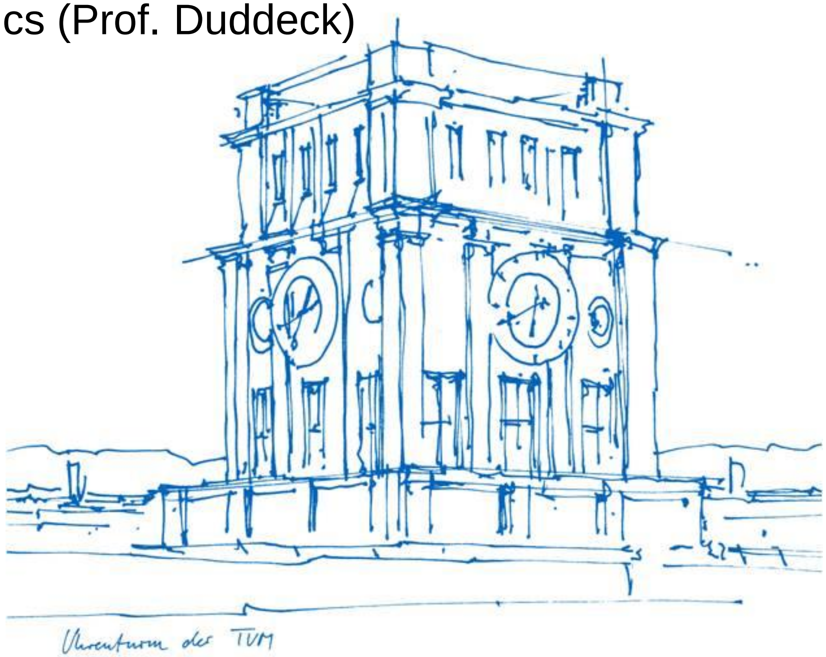
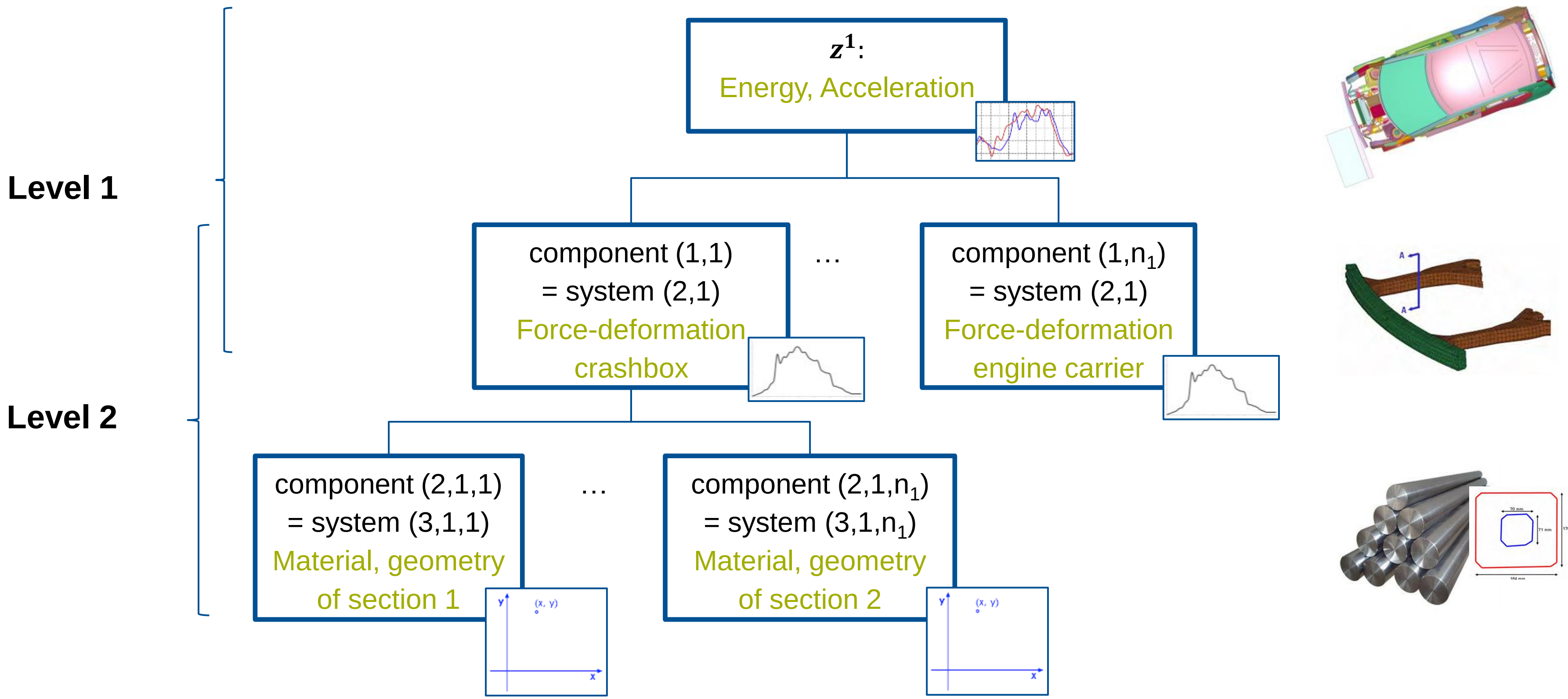


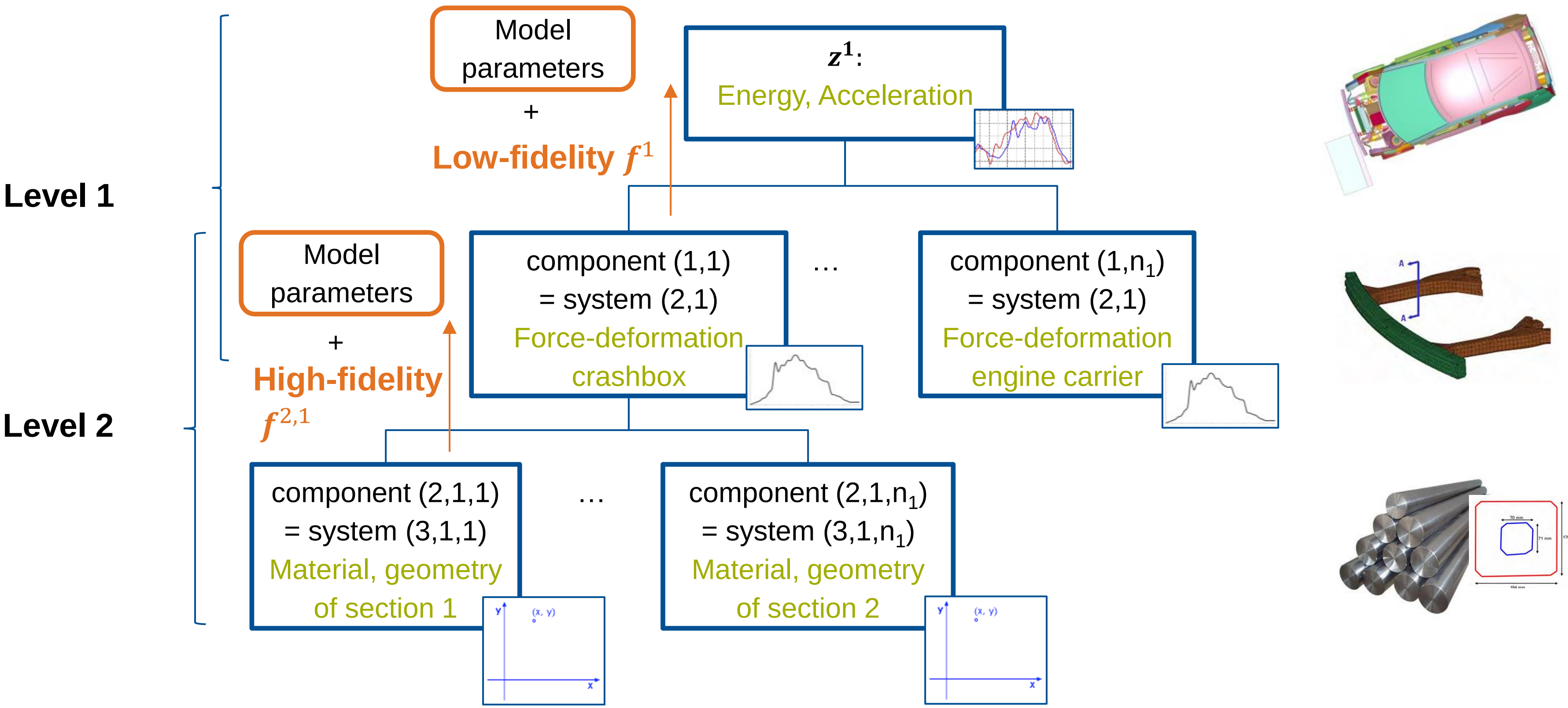
Organization

Motivation → Uncertainty modeling → **Methodology** → Application → Outlook

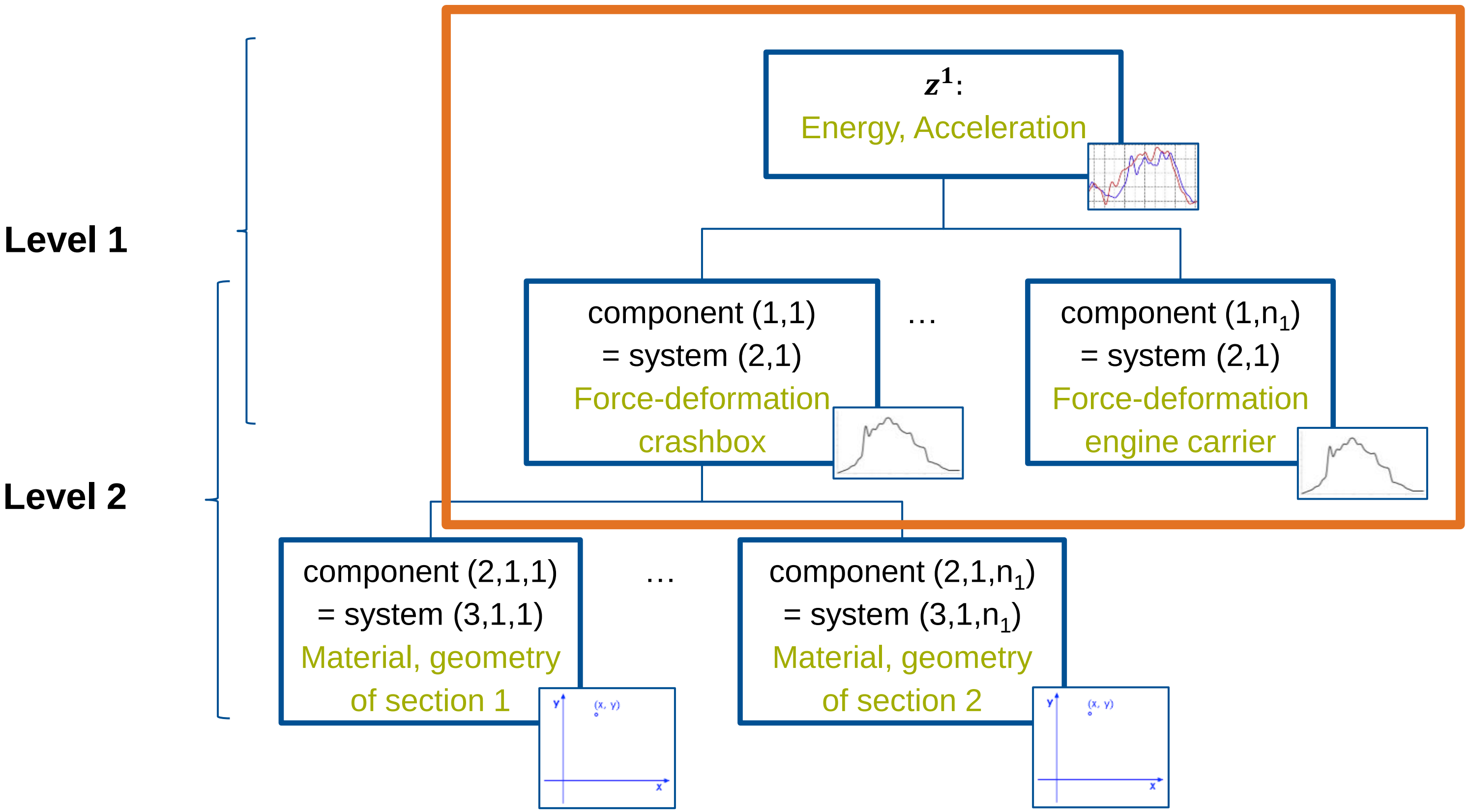
Example of a system for crash design

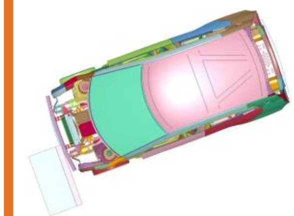


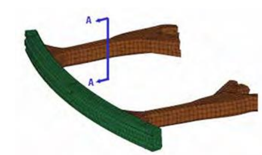
In early-phase crash design the system performances of level 2 are unknown

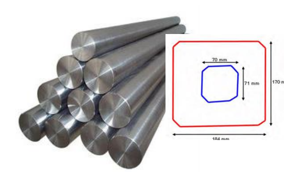


The focus is put on methods for level 1

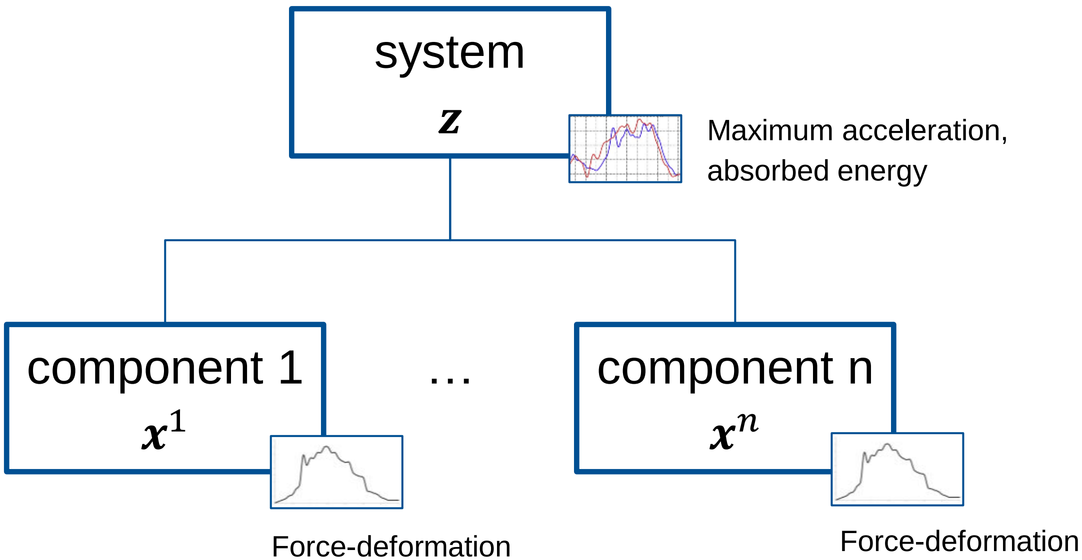




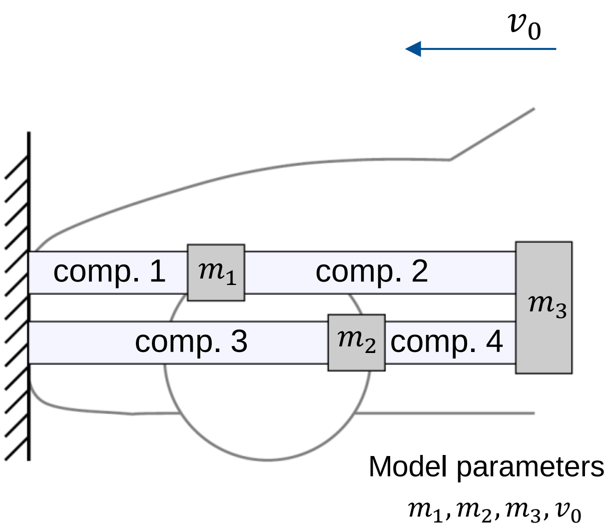




Crash design problem at a glance

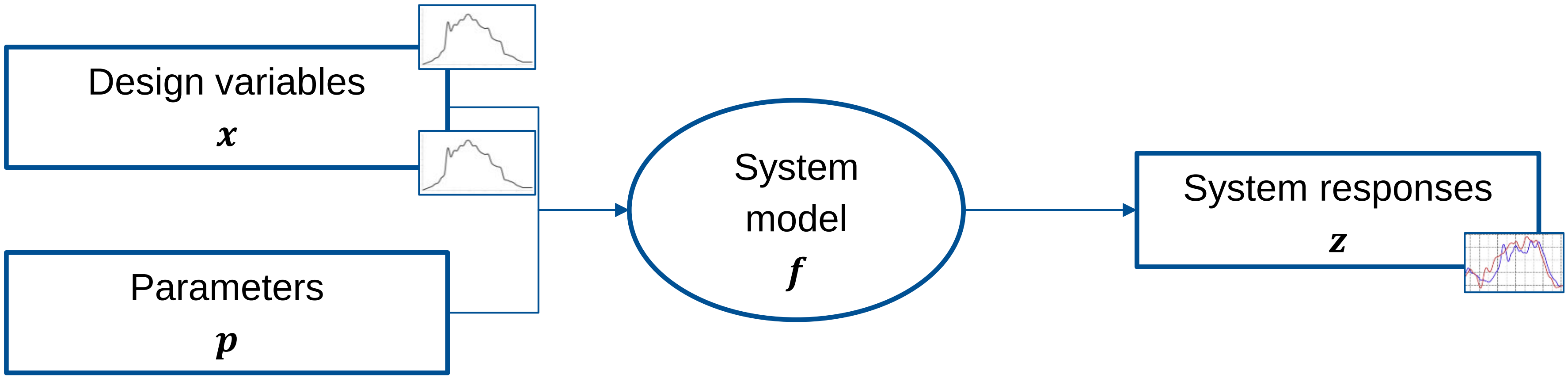


Geometry



Question: *"How to select the force-deformation characteristics of the components?"*

A functional view of the crash design problem



Lack-of-knowledge uncertainties are modeled as interval uncertainties here

At the early phase, design variables of level 1 are uncertain

- They are output quantities of level 2
- Very little information on level 2 is available

Modeled as interval uncertainties

$$\boldsymbol{x} \in [\breve{\boldsymbol{x}} - \boldsymbol{\delta}, \breve{\boldsymbol{x}} + \boldsymbol{\delta}]$$

Where: $\boldsymbol{\delta}$ is often unknown

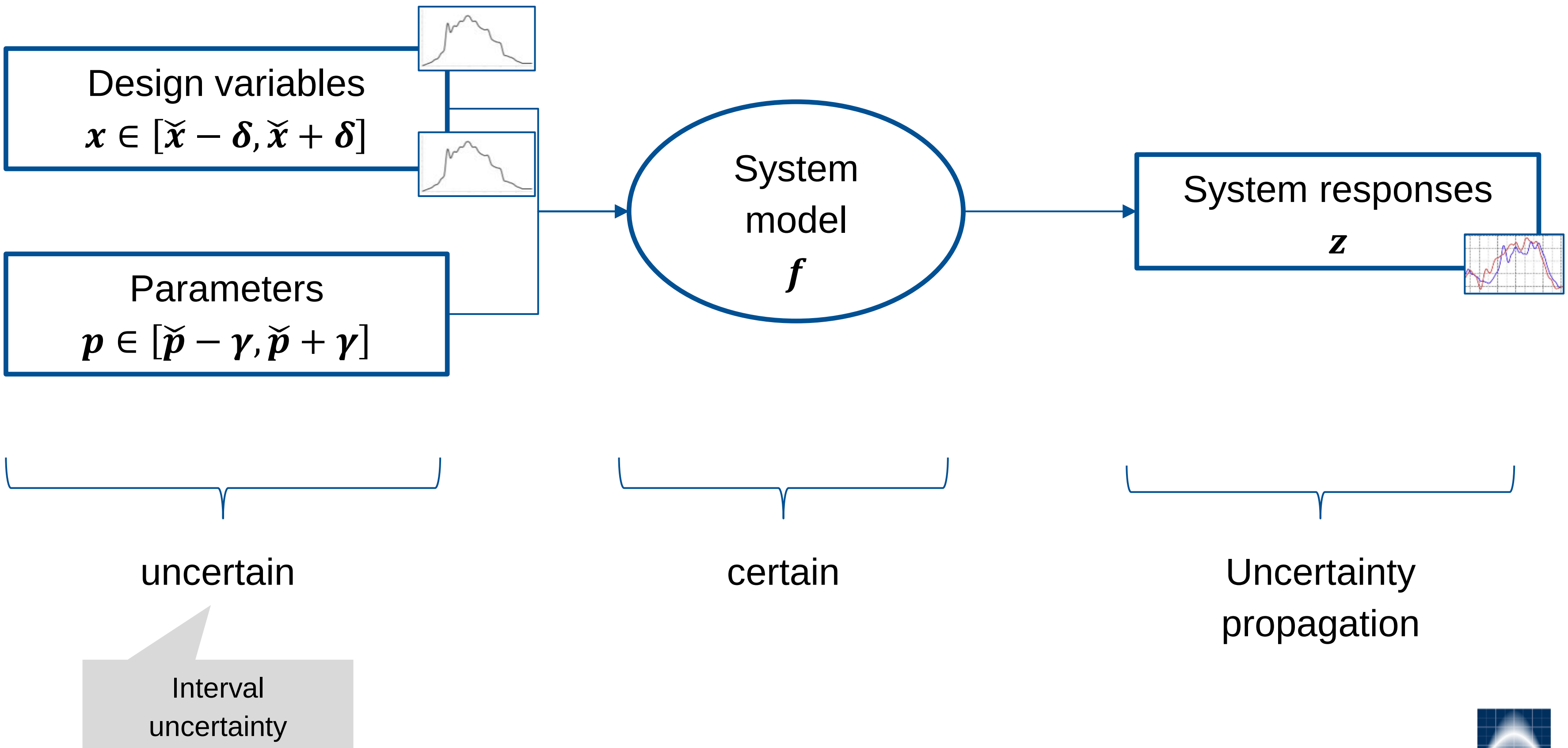
Real design $\boldsymbol{x}$
Intended design $\breve{\boldsymbol{x}}$

The same approach is taken for model parameters

$$\mathbf{p} \in [\breve{\boldsymbol{p}} - \boldsymbol{\gamma}, \breve{\boldsymbol{p}} + \boldsymbol{\gamma}]$$

Where: $\boldsymbol{\gamma}$ is usually known

Uncertainties at a glance



For known $\boldsymbol{\delta}$, robust optimization can be applied

Robust regularization [1]

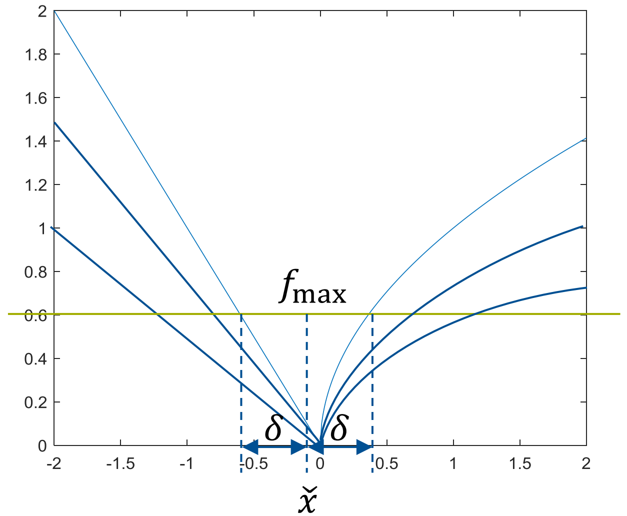
If we know $\boldsymbol{\delta}$, we can determine $\breve{\boldsymbol{x}}$ that minimize

$$\max_{\boldsymbol{x}} \{\boldsymbol{f}(\boldsymbol{x},\boldsymbol{p}) : \boldsymbol{x} \in [\breve{\boldsymbol{x}} - \boldsymbol{\delta}, \breve{\boldsymbol{x}} + \boldsymbol{\delta}],$$
$$\boldsymbol{p} \in [\breve{\boldsymbol{p}} - \boldsymbol{\gamma}, \breve{\boldsymbol{p}} + \boldsymbol{\gamma}] \ \}$$

Example modified from 1

$$f(x) = \begin{cases} -px, x < 0 \\ p\sqrt{x}, x \geq 0, \end{cases} \quad p \in [0.5,1]$$

$$\delta = 0.5$$



Source: [1]Beyer H. and Sendhoff B., 2007, „Robust optimization – a comprehensive survey“

We need a different approach here

Robust regularization [1]

If we know $\boldsymbol{\delta}$, we can determine $\breve{\boldsymbol{x}}$ that minimize

$$\max_{\boldsymbol{x}} \{\boldsymbol{f}(\boldsymbol{x},\boldsymbol{p}) : \boldsymbol{x} \in [\breve{\boldsymbol{x}} - \boldsymbol{\delta}, \breve{\boldsymbol{x}} + \boldsymbol{\delta}], \; \boldsymbol{p} \in [\breve{\boldsymbol{p}} - \boldsymbol{\gamma}, \breve{\boldsymbol{p}} + \boldsymbol{\gamma}] \;\}$$

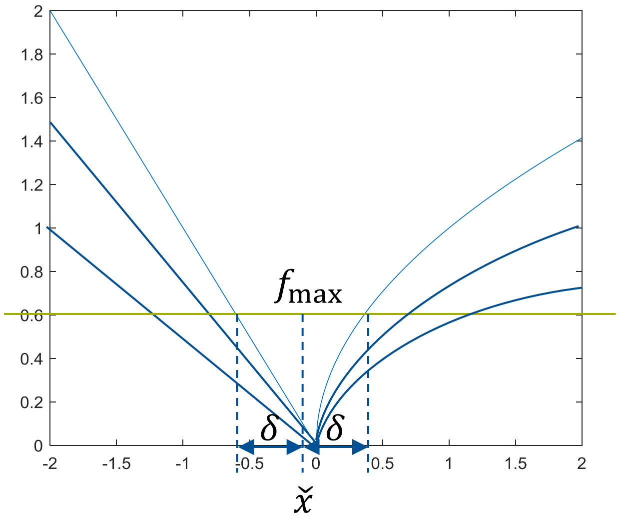
PROBLEM:

- We do not know $\boldsymbol{\delta}$
- No more „flexibility" in designing for level 2

Example modified from 1

$$f(x) = \begin{cases} -px, x < 0 \\ p\sqrt{x}, x \geq 0, \end{cases} \quad p \in [0.5,1]$$

$$\delta = 0.5$$



Source: [1]Beyer H. and Sendhoff B., 2007, „Robust optimization – a comprehensive survey"

Idea: Bounding the system responses

Bounding the all system responses by thresholds $\boldsymbol{f}_{\mathrm{c}}(\boldsymbol{p})$ instead of minimizing them.[2,3]

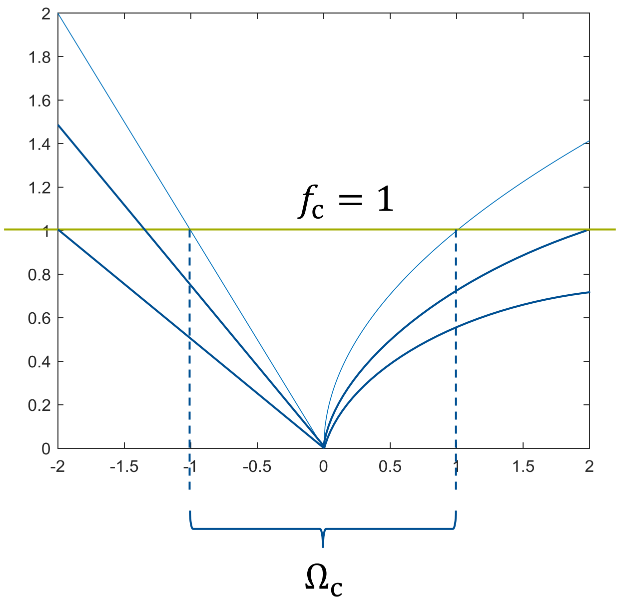
Idea also known from "Robust minimization" [1]

Obtain a set of permissible system designs for all parameters:

$$\Omega_{\mathrm{c}} = \{\boldsymbol{x} \in \Omega_{\mathrm{ds}}\colon \boldsymbol{f}(\boldsymbol{x},\boldsymbol{p}) \leq \boldsymbol{f}_{\mathrm{c}}(\boldsymbol{p}), \boldsymbol{p} \in [\breve{\boldsymbol{p}} - \boldsymbol{\gamma}, \breve{\boldsymbol{p}} + \boldsymbol{\gamma}]\}$$

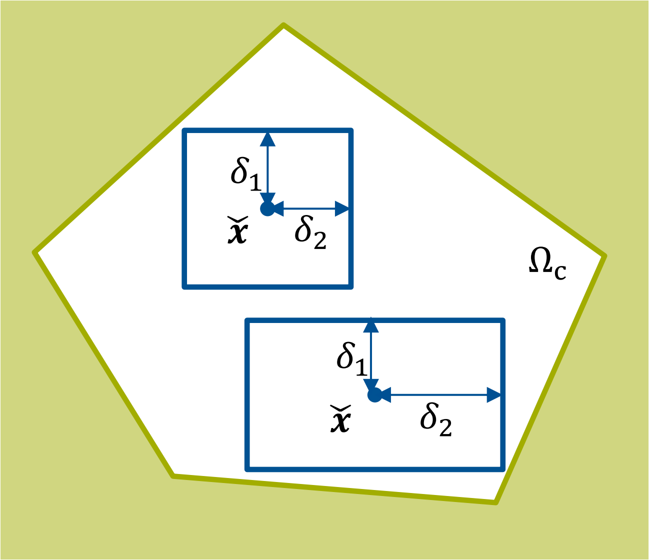
Example modified from 1

$$f(x) = \begin{cases} -px, x < 0 \\ p\sqrt{x}, x \geq 0, \end{cases} \quad p \in [0.5,1]$$



Source: [1]Beyer H-G & Sendhoff B. "Robust optimization – a comprehensive survey". CMAME 196(33–34):3190-3218 (2007).
[2]Rocco C.M. et al., 2003, „Robust design using a hybrid-cellular-evolutionary and interval-arithmetic approach“, Rel. Eng. & Sys. Saf. 79, 2 (2003), 149-159.
[3]Zimmermann M. and von Hoessle J.E., 2013, „Computing Solution Spaces for Robust Design“, Inter.. J. for Num. Meth. in Eng. 94, 3 (2013), 290-307.

Example: Candidates for $\boldsymbol{\delta}$, $\breve{\boldsymbol{x}}$



The bigger the rectangle formed by $\boldsymbol{\delta}, \breve{\boldsymbol{x}}$, i.e.

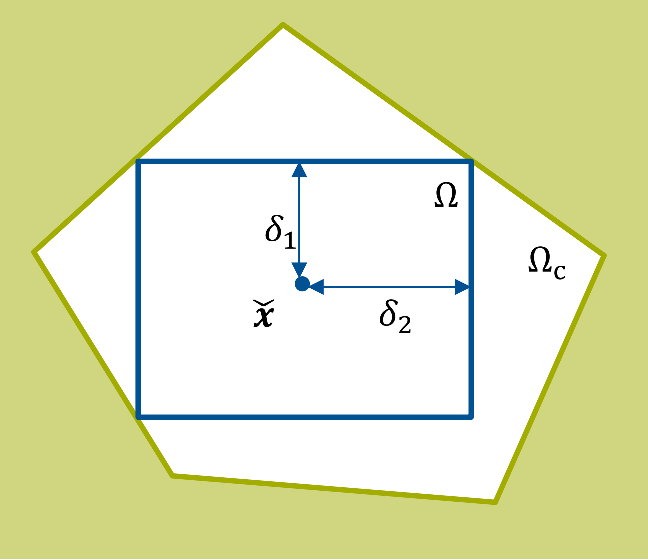
$$[\breve{\boldsymbol{x}} - \boldsymbol{\delta}, \breve{\boldsymbol{x}} + \boldsymbol{\delta}]$$

the larger uncertainties can be taken into account!

➡ If $\boldsymbol{\delta}$ is unknown, maximize its volume by varying $\boldsymbol{\delta}, \breve{\boldsymbol{x}}$

This largest rectangle can cope with lack-of-knowledge uncertainties

Example: largest rectangle



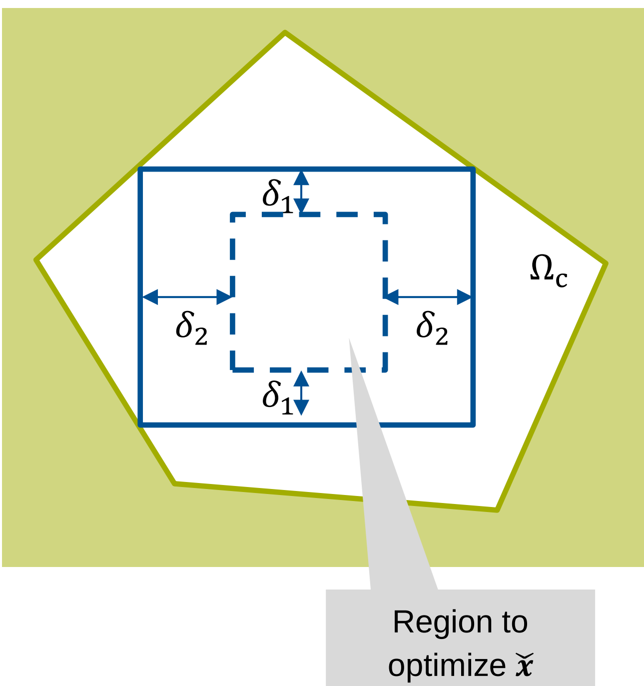
$\breve{\boldsymbol{x}}$ is defined as the *most robust design* as it allows the largest $\boldsymbol{\delta}$.[2] The rectangle itself is *(worst-case) box-shaped solution space*.[3]

$$\Omega = [\breve{\boldsymbol{x}} - \boldsymbol{\delta}, \breve{\boldsymbol{x}} + \boldsymbol{\delta}]$$

Source: [2]Rocco C.M. et al., 2003, „Robust design using a hybrid-cellular-evolutionary and interval-arithmetic approach“, Rel. Eng. & Sys. Saf. 79, 2 (2003), 149-159.
[3]Zimmermann M. and von Hoessle J.E., 2013, „Computing Solution Spaces for Robust Design“, Inter.. J. for Num. Meth. in Eng. 94, 3 (2013), 290-307.

This largest rectangle can cope with lack-of-knowledge uncertainties

Example: $\delta_1 = 0.1, \delta_2 = 0.2$



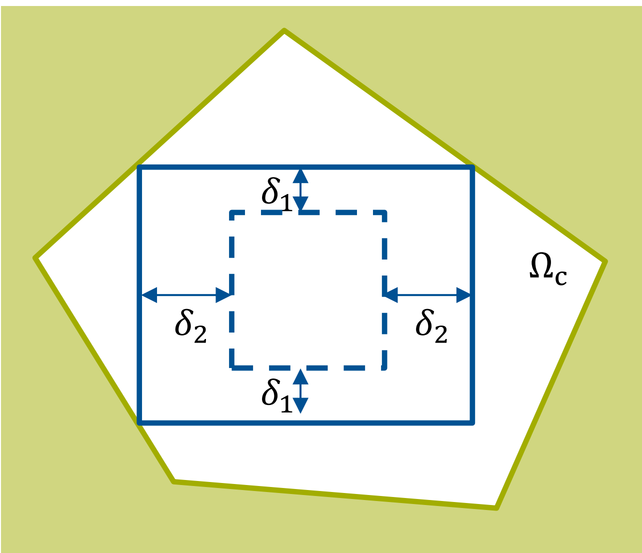
Later phase of systems design:

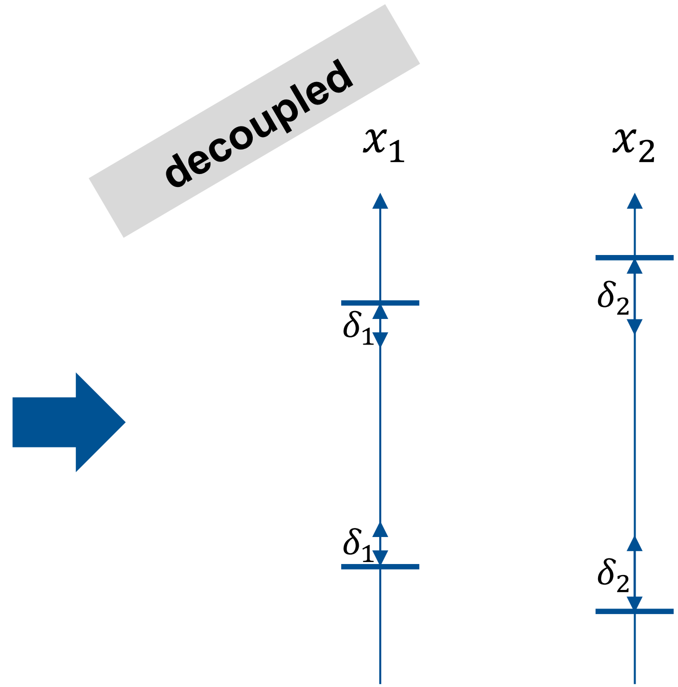
If it turns out that the entries of the real values of $\boldsymbol{\delta}$ are smaller than the ones of the largest rectangle…

- $\breve{\boldsymbol{x}}$ can also be optimized within the largest rectangle
- This can be done independently for every component!

Worst-case solution spaces are both an uncertainty and a product development tool

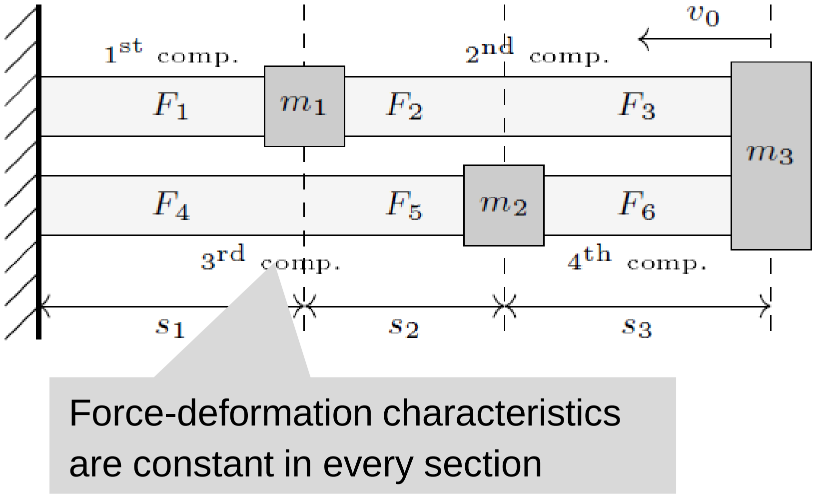
Example: $\delta_1 = 0.1, \delta_2 = 0.2$





A closer look to our crash design problem

Crash model for frontal car



Design variables

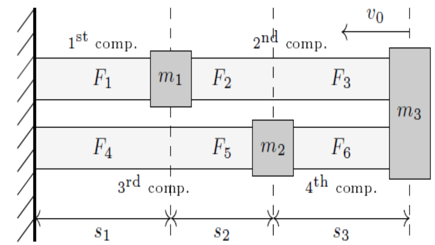
- $x = (F_1, F_2, F_3, F_4, F_5, F_6)$

Uncontrollable parameter	Interval of the real values
s_1	$[0.19\,\mathrm{m}, 0.21\,\mathrm{m}]$
s_2	$[0.14\,\mathrm{m}, 0.16\,\mathrm{m}]$
s_3	$[0.19\,\mathrm{m}, 0.21\,\mathrm{m}]$
m_1	$[95\,\mathrm{kg}, 105\,\mathrm{kg}]$
m_2	$[145\,\mathrm{kg}, 155\,\mathrm{kg}]$
m_3	$[1150\,\mathrm{kg}, 1250\,\mathrm{kg}]$
v_0	$[15.5\,\frac{\mathrm{m}}{\mathrm{s}}, 15.6\,\frac{\mathrm{m}}{\mathrm{s}}]$
a_c	$[290\,\frac{\mathrm{m}}{\mathrm{s}^2}, 310\,\frac{\mathrm{m}}{\mathrm{s}^2}]$

Table 1: Uncertainty modelling of the uncontrollable parameters

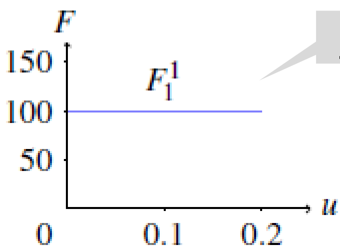
SPP 1886

The design variables of force-deformation characteristics



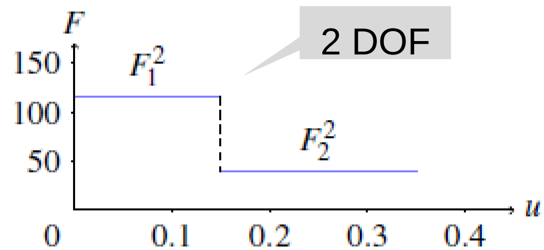
design example

1st component



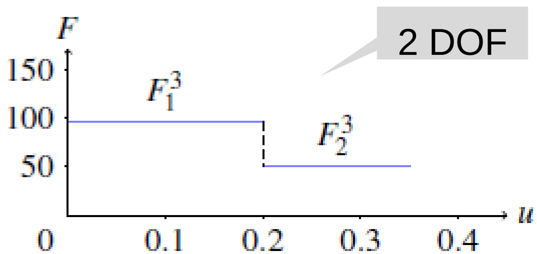
1 DOF

2nd component



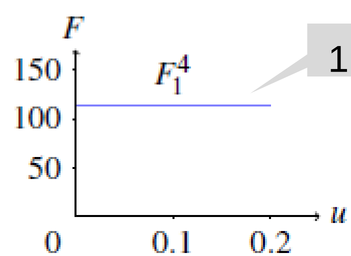
2 DOF

3rd component



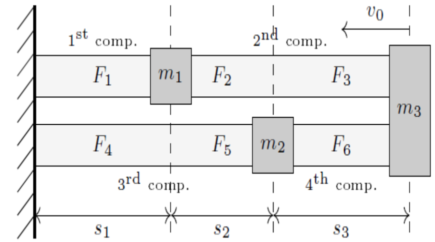
2 DOF

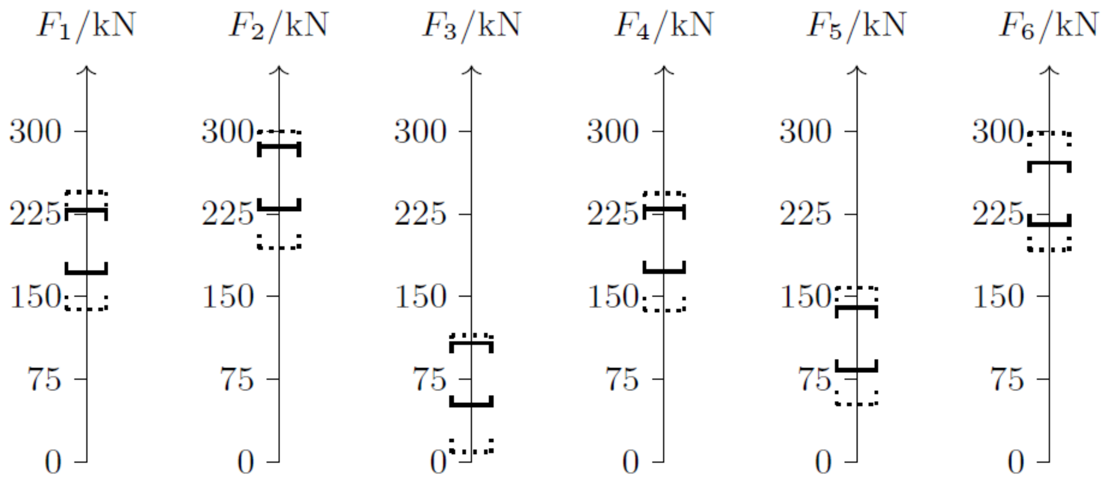
4th component



1 DOF

Results for the crash design problem





—— Worst-case solution space with $\boldsymbol{p} \in [\boldsymbol{p}^{\mathrm{l}}, \boldsymbol{p}^{\mathrm{u}}]$

------ Classical solution space with $\boldsymbol{p} = 0.5(\boldsymbol{p}^{\mathrm{u}} + \boldsymbol{p}^{\mathrm{l}})$

More methods and results are already available

Aspects that were already considered

- Uncertainty of system performances (interval type) ← WCCM, US, July2018
- Different shapes for $\Omega^1, \ldots, \Omega^n$ (example ellipsoids) ← ISUMA, BR, April2018

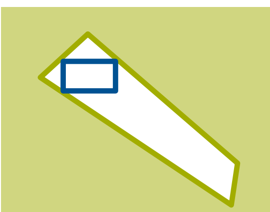
Outlook

- Generalize interval uncertainties to fuzzy uncertainties (where possible) ← Joint work with Uni Stuttgart
- Coupled worst-case solution spaces to avoid loss of solution space ← Joint work with BMW

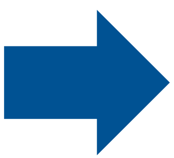
Thank you for your attention!

BACKUP

The largest rectangle can possibly cover only a small portion of permissible system designs



There are scenarios where the largest rectangle covers only a small portion of permissible system designs

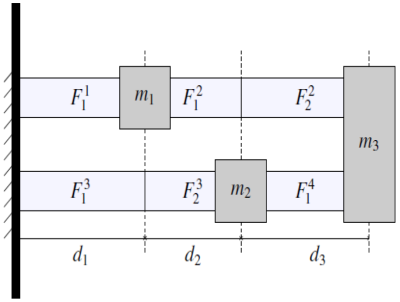


Need: Generalized approach for systems design that extends flexibility and toleration of variations

For a good crash test performance the system has to fulfill 3 relevant criteria

Maximum admissible acceleration

$$(F_1^1+F_1^3) \leq (m_1+m_2+m_3)a_c,$$
$$(F_1^2+F_2^3) \leq (m_2+m_3)a_c,$$
$$(F_2^2+F_1^4) \leq m_3 a_c$$



Energy absorption

$$-\frac{d_1}{m_1+m_2+m_3}(F_1^1+F_1^3) - \frac{d_2}{m_2+m_3}(F_1^2+F_2^3) - \frac{\mathrm{d}_3}{m_3}(F_2^2+F_1^4) \leq -\frac{1}{2}(v_0)^2$$

Order of deformation

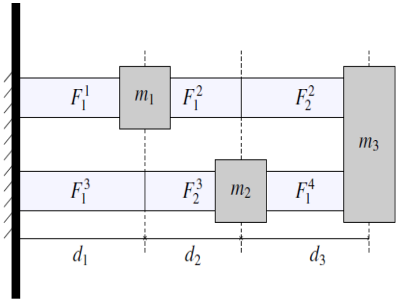
$$(1-\frac{m_1}{m_1+m_2+m_3})F_1^1 - \frac{m_1}{m_1+m_2+m_3}F_1^3 - F_1^2 \leq 0,$$
$$(1-\frac{m_1}{m_1+m_2+m_3})F_1^3 - \frac{m_1}{m_1+m_2+m_3}F_1^1 - F_1^4 \leq 0,$$
$$(1-\frac{m_2}{m_2+m_3})F_1^3 - \frac{m_2}{m_2+m_3}F_1^2 - F_1^4 \leq 0,$$

With these requirements the 6D complete solution space is formed

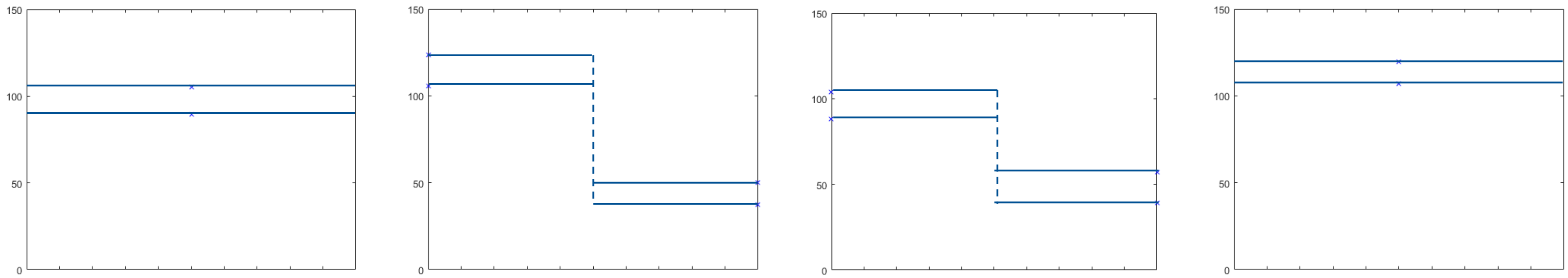
For crash design the solution spaces are lower/upper bounds for F-u-characteristics

Decomposed solution space

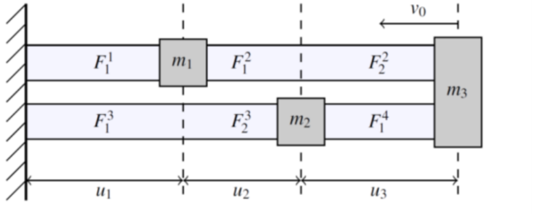
$$\Omega = [\mathrm{F}_1^{l,1}, \mathrm{F}_1^{l,1}] \times [\mathrm{F}_1^{l,2}, \mathrm{F}_1^{l,2}] \times [\mathrm{F}_2^{l,2}, \mathrm{F}_2^{l,2}]$$
$$\times [\mathrm{F}_1^{l,3}, \mathrm{F}_1^{l,3}] \times [\mathrm{F}_2^{l,3}, \mathrm{F}_2^{l,3}] \times [\mathrm{F}_1^{l,4}, \mathrm{F}_1^{l,4}]$$



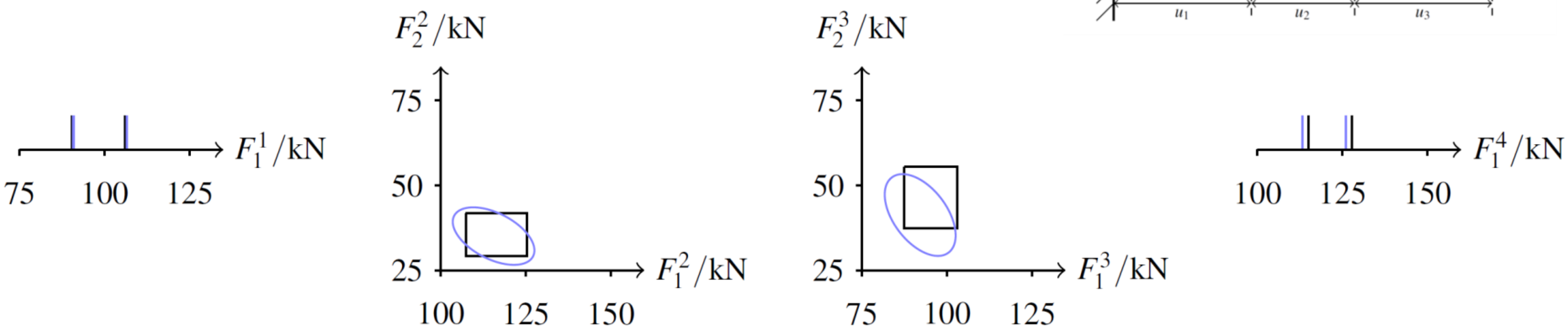
Visualization of decomposed solution space



For this crash design problem ellipsoids comprises a greater volume than rectangles



Largest sets for component design $\Omega^1, \ldots, \Omega^4$



smaller ← Volume (coverage of uncertainties and flexibility) → greater

Degrees of freedom for optimization (computationally expensive)

SPP 1886